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QUADRATIC DIVERGENCES IN GUTS WITH VANISHING ONE-LOOP BETA FUNCTIONS

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Abstract

All members of a recently proposed new set of (non-supersymmetric) grand unified theories with at the one-loop level vanishing beta functions for the gauge, Yukawa, and scalar-boson self-interaction coupling constants are shown to involve, already at the one-loop level, quadratically divergent contributions to both the vector-boson and scalar-boson masses.

1 Introduction

Supersymmetry rendered possible the construction of finite quantum field theories, i. e., quantum field theories which in their perturbation expansion are free from ultraviolet divergences, at least up to two loops [1] for (even softly broken [2]) $N = 1$ supersymmetric theories satisfying two so-called “finiteness conditions” and at all orders [3] for (again even softly broken [4]) $N = 2$ supersymmetric theories satisfying a single one-loop finiteness condition. The latter thus form a large class of finite quantum field theories [5] which includes the famous $N = 4$ super-Yang–Mills theory as a special case [6].

The discovery of these supersymmetric finite quantum field theories suggested to ask oneself whether supersymmetry is indeed a necessary prerequisite for finiteness in the sense described above [7, 8, 9] and, in particular, prompted the search for non-supersymmetric finite models. However, the conclusions at which one arrives unfortunately depend on the chosen regularization method: Dimensional regularization, for a fixed space–time dimension, ignores all quadratic divergences in the theory. In contrast to this, a regularization method which employs a dimensional regularization parameter like, e. g., cutoff regularization, allows to identify also this latter type of divergence, being thus by far more restrictive when demanding absence of divergences. Accordingly, all non-supersymmetric but “dimensional-regularization finite” models given so far [10] proved to entail quadratic divergences [11].

Very recently, some new set of models has been singled out by the requirement of vanishing one-loop β functions for all (gauge, Yukawa, and scalar-boson self-interaction) coupling constants in some massless theory with specific non-supersymmetric particle content [12].

In the present note we would like to demonstrate that the models proposed in Ref. [12], in spite of the fact that they do not possess any one-loop divergences when investigated by dimensional regularization, produce quadratically divergent one-loop contributions to the masses of both the vector bosons and the scalar bosons present in the theory. To this end we discuss, in Sect. 2, the quadratic divergences arising in a general gauge theory at the one-loop level. In Sect. 3 we review the one-loop finite models of Ref. [12] and, by application of the previous general discussion, analyse these models with respect to their eventual quadratic divergences. We are forced to conclude, in Sect. 4, that all of these models, already at one-loop level, involve quadratic divergences.

2 Quadratic Divergences in a General Gauge Theory

A gauge theory is characterized by invariance with respect to local transformations forming some compact—in general, non-Abelian—gauge group \mathcal{G} , which is defined in terms of its (completely antisymmetric) structure constants f_{abc} by the commutation relations $[T^a, T^b] = if_{abc}T^c$ of its (Hermitean) generators T^a . Upon ignoring all dimensional parameters like masses and cubic self-couplings of scalar bosons (which do not affect the high-energy behaviour of the theory), the Lagrangian defining a general gauge theory for

- (Hermitean) vector gauge fields V_μ^a in the adjoint representation G of \mathcal{G} ,
- two-component Weyl spinor fields $\psi_{iL} \equiv \frac{1}{2}(1 + \gamma_5)\psi_i$ in some fermion representation F of \mathcal{G} (because of $(\psi_R)^c = \psi^c_L$ without loss of generality all of them assumed to be, say, left-chiral), and
- Hermitean scalar fields ϕ_m in some (necessarily) real scalar-boson representation S of \mathcal{G}

is given by

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + i\overline{\psi_L} \gamma^\mu D_\mu \psi_L + \frac{1}{2}(D_\mu \phi)^T D^\mu \phi \\ & + \frac{1}{2}[\overline{\psi_R^c} h^m \psi_L \phi_m + \text{H. c.}] - \frac{1}{4!} \lambda_{mnpq} \phi_m \phi_n \phi_p \phi_q \quad , \end{aligned} \quad (1)$$

where $F_{\mu\nu}^a$ denotes the gauge-covariant field strength

$$F_{\mu\nu}^a \equiv \partial_\mu V_\nu^a - \partial_\nu V_\mu^a + g f_{abc} V_\mu^b V_\nu^c \quad (2)$$

and D_μ the gauge-covariant derivative

$$D_\mu \equiv \partial_\mu - ig V_\mu^a T_R^a \quad , \quad R = F, S \quad , \quad (3)$$

both of them introducing gauge interactions with coupling strength g . Permutation symmetry and gauge invariance impose some obvious restrictions on the Yukawa coupling matrices h_{ik}^m as well as on the quartic self-couplings λ_{mnpq} of the scalar bosons.

On dimensional grounds, for the above theory quadratic divergences can only arise in the masses of vector bosons—which would destroy gauge invariance—and in the masses of scalar bosons. Apart from some

trivial factors, these quadratically divergent contributions to vector and scalar-boson masses are proportional to some quantities Q_V and Q_S , respectively, which at the one-loop level, when expressed in terms of the quadratic Casimir operator

$$\sum_{\sigma} C_2(R_{\sigma}) E_{ik}^{\sigma} := (T_R^a T_R^a)_{ik} \quad (4)$$

(where E^{σ} denotes the projector onto the irreducible component R_{σ} in the decomposition $R = \bigoplus_{\sigma} R_{\sigma}$ of the maybe reducible representation R) and the second-order Dynkin index

$$T(R)\delta_{ab} := \text{Tr}(T_R^a T_R^b) \quad , \quad T(R) = \sum_{\sigma} T(R_{\sigma}) \quad , \quad (5)$$

read for the mass of the vector bosons [7]

$$Q_V = 2 C_2(G) - 2 T(F) + T(S) \quad (6)$$

and for the mass of the scalar bosons [7, 8, 11]

$$(Q_S)_{mn} = 6 g^2 (T_S^a T_S^a)_{mn} - 4 \text{Re} \text{ Tr} (h^m h^{n\dagger}) + \lambda_{mnpp} \quad . \quad (7)$$

In the next section we show that all models given in Ref. [12], although possessing vanishing one-loop β functions, yield non-vanishing values for the quantities Q_V and Q_S , and thus have to be regarded as merely “pseudo-finite”.

3 The Pseudo-Finite Models

Both of the models under consideration are based on the gauge group $\mathcal{G} = \text{SU}(N)$ and involve only particles transforming either according to the fundamental representation R_f (of dimension $d_f = N$) or according to the adjoint representation G (of dimension $d_G = N^2 - 1$) of \mathcal{G} . In order to be able to fulfill the requirements of the assumed gauge invariance, both models have to contain (real) gauge vector bosons V_{μ}^a , $a = 1, 2, \dots, N^2 - 1$, transforming, of course, according to the adjoint representation G of the gauge group \mathcal{G} , i. e.,

$$V_{\mu} \sim G \quad . \quad (8)$$

As a consequence, in both models all couplings may be expressed in terms of the generators $(T_G^a)_{bc} = \frac{1}{i} f_{abc}$ in the adjoint representation G of \mathcal{G} , the generators T_f in the fundamental representation R_f of \mathcal{G} , or the completely symmetric constants $d_{abc} \equiv \text{Tr} (\{T_f^a, T_f^b\} T_f^c) / T(R_f)$.

3.1 The general model

The non-vector particle content of this model consists of

- m sets of Dirac fermions $\Psi_{(k)}$, $k = 1, 2, \dots, m$, each of these sets transforming according to the adjoint representation G of \mathcal{G} , i. e.,

$$\Psi_{(k)} \sim G \quad , \quad k = 1, 2, \dots, m \quad ; \quad (9)$$

- m sets of Dirac fermions $\chi_{(k)}$, $k = 1, 2, \dots, m$, each of these sets transforming according to the fundamental representation R_f of \mathcal{G} , i. e.,

$$\chi_{(k)} \sim R_f \quad , \quad k = 1, 2, \dots, m \quad ; \quad (10)$$

- n sets of Dirac fermions $\zeta_{(k)}$, $k = 1, 2, \dots, n$, each of these sets transforming according to the fundamental representation R_f of \mathcal{G} , i. e.,

$$\zeta_{(k)} \sim R_f \quad , \quad k = 1, 2, \dots, n \quad ; \quad (11)$$

- real scalar bosons Φ^a , $a = 1, 2, \dots, N^2 - 1$, transforming according to the adjoint representation G of \mathcal{G} , i. e.,

$$\Phi \sim G \quad ; \quad (12)$$

- (necessarily) complex scalar bosons φ transforming according to the fundamental representation R_f of \mathcal{G} , i. e.,

$$\varphi \sim R_f \quad . \quad (13)$$

The Lagrangian defining this general model reads [12]

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + i \sum_{k=1}^m \bar{\Psi}_{(k)}^a (\not{D}_{ab} - h_1 f_{abc} \Phi^c) \Psi_{(k)}^b \\ & + i \sum_{k=1}^m \bar{\chi}_{(k)} (\not{D} - i h_2 T_f^a \Phi^a) \chi_{(k)} + i \sum_{k=1}^n \bar{\zeta}_{(k)} \not{D} \zeta_{(k)} \\ & + \left(i h_3 \sum_{k=1}^m \bar{\chi}_{(k)} T_f^a \varphi \Psi_{(k)}^a + \text{H. c.} \right) \\ & + \frac{1}{2} (D_\mu \Phi)^T D^\mu \Phi + (D_\mu \varphi)^\dagger D^\mu \varphi - \frac{\lambda_1}{8} (\Phi^T \Phi)^2 - \frac{\lambda_2}{8} (\Phi^a d_{abc} \Phi^b)^2 \\ & - \frac{\lambda_3}{2} (\Phi^T \Phi) (\varphi^\dagger \varphi) - \frac{\lambda_4}{2} (\Phi^a d_{abc} \Phi^b) (\varphi^\dagger T_f^c \varphi) - \frac{\lambda_5}{2} (\varphi^\dagger \varphi)^2 \quad . \quad (14) \end{aligned}$$

The fermions $\chi_{(k)}$ are discriminated from the fermions $\zeta_{(k)}$ by the fact that the former also undergo Yukawa interactions whereas the latter do not.

Finiteness of the one-loop contribution to the renormalization of the gauge coupling constant, as expressed by the relation

$$21N - 4[(2N + 1)m + n] = 1 \quad , \quad (15)$$

restricts the possible gauge groups $SU(N)$ to the values $N = 4\ell + 1$ for $\ell = 1, 2, \dots$. The multiplicities m and n allowed by Eq. (15) for the groups $SU(5)$ and $SU(9)$ are listed, together with the respective number of solutions¹ of the one-loop finiteness conditions for Yukawa interactions and quartic scalar-boson self-couplings, in Table 1.

Table 1: Multiplicities m and n allowed by one-loop finiteness of the gauge coupling constant for general models based on the smallest conceivable gauge groups $SU(5)$ and $SU(9)$, and corresponding number of solutions of the one-loop finiteness conditions for the Yukawa interactions and quartic scalar-boson self-couplings

N	m	n	number of solutions
5	0	26	0
	1	15	1
	2	4	0
9	0	47	0
	1	28	1
	2	9	1

The numerical values of the Yukawa coupling constants h_1, h_2, h_3 and of the scalar-boson self-coupling constants $\lambda_1, \lambda_2, \dots, \lambda_5$ which render the three models filtered out by the analysis in Ref. [12] finite at the one-loop level are compiled in Table 2.

It is, however, an easy task to convince oneself that for the present general model the quantities Q_V and Q_S as defined in Eqs. (6) and (7), which parametrize the magnitude of the one-loop contribution to

¹ By inspection, the $(N = 5, m = 2, n = 4)$ model of Ref. [12], although attributed by the authors to be one-loop finite, turns out—at least for the numerical values of the Yukawa interactions and quartic scalar-boson self-couplings given in Ref. [12]—to possess non-vanishing one-loop β functions and thus to be not even pseudo-finite.

Table 2: Numerical values [12] of the Yukawa coupling constants h_i^2 , $i = 1, 2, 3$, and of the scalar-boson self-coupling constants λ_i , $i = 1, 2, \dots, 5$, for the general pseudo-finite models identified in Table 1 as well as the resulting values of the quantity Q_S , which parametrizes the magnitude of the quadratically divergent one-loop contribution to the scalar-boson masses, for both the sectors of the scalar bosons Φ and φ , denoted in these sectors by $Q_S^{(\Phi)}$ and $Q_S^{(\varphi)}$, respectively. (All of these quantities in units of g^2 .)

Model	$N = 5$ $m = 1, n = 15$	$N = 9$ $m = 1, n = 28$	$N = 9$ $m = 2, n = 9$
h_1^2/g^2	1.4211...	1.4546...	0.9817...
h_2^2/g^2	1.6806...	1.7416...	0.3878...
h_3^2/g^2	2.3612...	2.3294...	1.1273...
λ_1/g^2	0.6594...	0.4149...	0.3685...
λ_2/g^2	1.2933...	1.1947...	0.6880...
λ_3/g^2	0.3235...	0.1756...	0.0899...
λ_4/g^2	1.6765...	1.7329...	0.9858...
λ_5/g^2	1.0385...	1.1369...	0.6088...
$Q_S^{(\Phi)}/g^2$	-2.32	-0.07	-46.86
$Q_S^{(\varphi)}/g^2$	-10.71	-19.37	-34.13

the quadratic divergence of the vector-boson and scalar-boson masses, respectively, are definitely non-vanishing:

- Q_V is given by

$$Q_V = 3N - 2[(2N + 1)m + n] + 1 \quad (16)$$

or—after elimination of the fermion contribution with the help of the one-loop finiteness condition (15) for the gauge coupling constant—by

$$Q_V = -\frac{15N - 3}{2} \quad , \quad (17)$$

which is beyond doubt unequal to zero for any integer N and, in fact, strictly negative for all $N = 1, 2, \dots$

- Q_S is given in the sector of the scalar bosons Φ by

$$\begin{aligned} Q_S^{(\Phi)} = & 6Ng^2 - 4m(2Nh_1^2 + h_2^2) \\ & + (N^2 + 1)\lambda_1 + 2\frac{N^2 - 4}{N}\lambda_2 + 2N\lambda_3 \end{aligned} \quad (18)$$

and in the sector of the scalar bosons φ by

$$\begin{aligned} Q_S^{(\varphi)} = & 3\frac{N^2 - 1}{N}g^2 - 4m\frac{N^2 - 1}{N}h_3^2 \\ & + (N^2 - 1)\lambda_3 + 2(N + 1)\lambda_5 . \end{aligned} \quad (19)$$

Evaluation of the right-hand sides of Eqs. (18) and (19) with the help of the three sets of solutions for the Yukawa interactions and scalar-boson self-couplings quoted in Table 2 yields the numerical results for $Q_S^{(\Phi)}$ and $Q_S^{(\varphi)}$ given also in Table 2. The non-vanishing values of these quantities indicate unambiguously that in each of the general one-loop pseudo-finite models of Table 1 there arise quadratic divergences for the masses of the scalar bosons.

3.2 The simplified model

This model is obtained from the more general model described above by completely decoupling the fermions $\Psi_{(k)}$, Eq. (9), and $\zeta_{(k)}$, Eq. (11), as well as the scalar bosons φ , Eq. (13), from the theory. Accordingly, the non-vector particle content of this model consists of

- m sets of Dirac fermions $\chi_{(k)}$, $k = 1, 2, \dots, m$, each of these sets transforming according to the fundamental representation R_f of \mathcal{G} , i. e.,

$$\chi_{(k)} \sim R_f \quad , \quad k = 1, 2, \dots, m ; \quad (20)$$

- real scalar bosons Φ^a , $a = 1, 2, \dots, N^2 - 1$, transforming according to the adjoint representation G of \mathcal{G} , i. e.,

$$\Phi \sim G . \quad (21)$$

Consequently, the Lagrangian defining this simplified model reads

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} + i \sum_{k=1}^m \bar{\chi}_{(k)} (\not{D} - ihT_f^a \Phi^a) \chi_{(k)} \\ & + \frac{1}{2} (D_\mu \Phi)^T D^\mu \Phi - \frac{\lambda_1}{8} (\Phi^T \Phi)^2 - \frac{\lambda_2}{8} (\Phi^a d_{abc} \Phi^b)^2 . \end{aligned} \quad (22)$$

Finiteness of the one-loop contribution to the renormalization of the gauge coupling constant, as expressed in this case by the relation

$$21N = 4m \quad , \quad (23)$$

now restricts the possible gauge groups $SU(N)$ to the values $N = 4\ell$ for $\ell = 1, 2, \dots$. The multiplicity m fixed by Eq. (23) for the groups $SU(4)$ and $SU(8)$ is listed, together with the respective number of solutions of the one-loop finiteness conditions for Yukawa interactions and quartic scalar-boson self-couplings, in Table 3. According to this, there exist two solutions for the $(N = 8, m = 42)$ model whereas there are none for the $(N = 4, m = 21)$ model. Both of these solutions are characterized by vanishing Yukawa interactions, i. e., by $h = 0$ [12].

Table 3: Multiplicity m as fixed by one-loop finiteness of the gauge coupling constant for simplified models based on the smallest conceivable gauge groups $SU(4)$ and $SU(8)$, and corresponding number of solutions of the one-loop finiteness conditions for the Yukawa interactions and quartic scalar-boson self-couplings

N	m	number of solutions
4	21	0
8	42	2

Again it is straightforward to check whether or not the quantities Q_V and Q_S , which characterize the one-loop quadratic divergences in vector- and scalar-boson masses, respectively, vanish:

- Q_V reduces from Eq. (16), valid for the general model (14), to

$$Q_V = 3N - 2m \quad (24)$$

or—when replacing the multiplicity m by the expression resulting from the one-loop finiteness condition (23) for the gauge coupling constant—to

$$Q_V = -\frac{15N}{2} \quad , \quad (25)$$

which obviously is strictly negative for all, in any case positive, N .

- Without any further calculation, Q_S may be read off immediately from the corresponding expression $Q_S^{(\Phi)}$ for the scalar bosons Φ of the general model, given in Eq. (18), by just dropping those contributions which arise, on the one hand, from the Yukawa coupling proportional to h_1 and, on the other hand, from the scalar-boson self-interaction proportional to λ_3 (and by re-labelling h_2 simply by h):

$$(Q_S)_{ab} =: Q_S \delta_{ab} \quad (26)$$

with

$$Q_S = 6Ng^2 - 4mh^2 + (N^2 + 1) \lambda_1 + 2 \frac{N^2 - 4}{N} \lambda_2 \quad . \quad (27)$$

Since, for reasons of stability of the theory, all λ_i , $i = 1, 2$, have to be positive, this last expression is for $h = 0$, irrespective of the precise numerical values of the couplings λ_i , strictly positive.

4 Conclusion

Two recently proposed sets of grand unified theories, characterized by a definite choice of some specific non-supersymmetric particle content and the fact that the beta functions of their gauge, Yukawa, and scalar-boson self-interaction coupling constants vanish at the one-loop level, have been investigated with respect to the eventual appearance of quadratic divergences in the course of renormalization of vector-boson and scalar-boson masses, respectively. Both of these two sets of models are based on the gauge group $SU(N)$; the more general one involves fermions and scalar bosons in (some multiples of) the fundamental and the adjoint representation of $SU(N)$, the rather simplified one contains only fermions in some multiple of the fundamental representation and scalar bosons in the adjoint representation of $SU(N)$. The requirement of vanishing one-loop beta functions fixes the possible gauge groups, i. e., N , the multiplicities of all the fermion representations, as well as the numerical values of the Yukawa and scalar-boson self-interaction coupling constants. The resulting models thus appear to be one-loop finite when all divergences are handled by dimensional regularization.

This situation, however, may change completely when employing a regularization method which operates with a dimensional regulator. And indeed, in the present analysis we were able to show that in each of the above models both the vector-boson and scalar-boson masses

receive quadratically divergent contributions at the one-loop level. In other words, all of these models are plagued by quadratic divergences and, consequently, should not be regarded to be one-loop finite in a regularization-scheme independent manner.

Moreover, by considering the condition for two-loop finiteness of the gauge coupling constant [7, 8, 10], all of the above models can easily be shown to loose their pseudo-finiteness at the two-loop level, as has been suspected already by their authors themselves [12].

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